

Name of Course	: <b>Generic Elective</b>
Unique Paper Code	: <b>32355301</b>
Name of Paper	: <b>GE-3 Differential Equations</b>
Semester	: <b>III</b>
Duration	: <b>3 hours</b>
Maximum Marks	: <b>75 Marks</b>

*Attempt any four questions. All questions carry equal marks.*

1. Determine the constant  $A$  such that the differential equation

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

is exact and solve the resulting exact equation.

Solve the following initial value problems:

- i)  $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, y(2) = -1, x > 0.$   
 ii)  $(3xy + y^2)dx + (x^2 + xy)dy = 0, y(1) = -1.$

2. Find the orthogonal trajectory of the family  $y = c \sin x$  that passes through the point  $(2\pi, 2)$ . Also find the family of oblique trajectory that intersects the family of circles  $x^2 + y^2 = c^2$  at an angle  $\frac{\pi}{4}$ .

Show that the family of confocal conics  $\frac{x^2}{\lambda+a^2} + \frac{y^2}{\lambda+b^2} = 1$ , where  $a$  and  $b$  are fixed constants and  $\lambda$  is the parameter, is self orthogonal.

3. Show that the set  $\{1, x, x^2\}$  of functions forms a basis for the solution set of a differential equation. Also, find such a differential equation.

Find the general solution of the second order equation  $t^2y'' + 2ty' - 2y = 0$  given that  $y_1(t) = t$  is a solution. Also solve the initial value problem

$$y''' + 3y'' - 4y = 0, y(0) = 1, y'(0) = 0, y''(0) = \frac{1}{2}.$$

4. Find the general solution of the following differential equations:

- i)  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 3e^{2x}.$   
 ii)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x.$   
 iii)  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x.$

5. Find the partial differential equation arising from the equation  $ax^2 + by^2 + z^2 = 1$ , where  $z = z(x, y)$ .

Find the general solution of the linear partial differential equation

$$(y + xu)p - (x + yu)q = x^2 - y^2.$$

Using  $v = \ln u$  and  $v = f(x) + g(y)$ , solve the equation

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \quad u(x, 0) = e^{x^2}.$$

6. Reduce the following equations to canonical form and hence find their solutions

- i)  $u_x - yu_y = 1 + u.$
- ii)  $y u_{xx} + (x + y)u_{xy} + x u_{yy} = 0, \quad y \neq x.$
- iii)  $u_{xx} - 4u_{xy} + 4u_{yy} = 0.$

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